Implementing Motivational Feedback with Entropy Control in Neural Networks

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Abstract

The theory of building artificial intelligence is reviewed including both biological and informational concepts. Based upon conclusions from Gödel's Incompleteness Theorem and Hilbert's 2^{nd} Problem, building a hierarchical system is proposed. Each level of the system has different plasticity, and components of one level are partially duplicated in the levels above. The system is built using a neural network and is referred to as an inductive automaton, which is an emulation of a closed physical system. Related physical laws are then studied and Boltzmann's entropy is proved to be anthropomorphic. Further it is proposed to use entropy to discover "unexpected truths" while groups of neurons combining various knowledge units are in the "hi-dissipation" state of energy consumption which corresponds to Boltzmann's entropy being at its maximum. To complete inductive automaton's architecture, Emotion Centre is introduced as a mechanism of neural network activity modulation. Gnostic neuron's memory function is treated as a general phenomenon with memorizing being a mechanism of self-organization of inductive automaton, suggesting that inductive automaton must solve one basic task – memorizing of external and internal information flow.

1 Self-organization

The general principle of self-organization of information has already proved useful in various pattern recognition and classification tasks. Self-organizing neural networks draw inspiration from the functionality of biological neurons and the brain, which might suggest their usability also in producing higher, human-like forms of intelligence [Fausset, 1995], [Allinson, 1992]. Typical neural networks however, are really not 'bio-logical' or organic in their basic network architecture or functionality. They are rather hardwired structures with adaptive connections between the modeled 'neurons'.

Since there are few working algorithms or implementations for the self-organization of formalized organic systems, in this paper we chart the possibilities of building AI systems based on genuinely organic formalized models and consider the physical and philosophical nature and implications of such systems.

The formal foundations of organic systems have been studied in detail by Chilean biologists Humberto Maturana and Francisco Varela [Maturana and Varela, 1987] under the general topic of Autopoietic Theory. "An autopoietic machine is a machine organized (defined as a unity) as a network of processes of production (transformation and destruction) of components that produce the components which: (i) through their interactions and transformations continuously regenerate and realize the network of processes (relations) that produced them; and (ii) constitute it (the machine) as a concrete unity in the space in which they (the components) exist by specifying the topological domain of its realization as such a network [Maturana and Varela, 1980]". Cells, for example, keep producing protein components that know how to combine with each other to sustain both the external border of the cell and its internal component factory – the main product of the factory is the factory itself, engines and walls included.

In the case of the human brain and its neural signaling the role of self-reference is obvious. Only about 1 % of the estimated 100 billion neurons of the brain are receiving direct signals from sensory organs or sending out direct signals to the muscles [Hebb, 1949]. Most neural pathways and synaptic connections are engaged in the internal communications of the brain. The self-referential network topology is further reflected in the signal dynamics of the brain: the dynamic states of the brain can be considered to be mainly a product of pre-existing dynamic states of the brain, not of the immediate sensory input.

Attempts to describe principles of self-referential algorithmic base were given the name of Hilbert's 2^{nd} Problem. Alan Turing concluded in research on his machine that it is impossible to use algorithms of some formal logical system for control of the system itself, produced by given algorithms. Later Gödel's Incompleteness Theorem gave a more general outlook to the problem.

According to Gödel's Theorem (mathematical definition of the theorem), within any given branch of mathematics, there would always be some propositions that couldn't be proven either true or false using rules and axioms of that mathematical branch itself. The implication is that all logical systems of any complexity are, by definition, incomplete; each of them contains, at any given time, more true statements than it can possibly prove according to its own defining set of rules. Gödel's Theorem suggests that a computer can never be as smart as human being because of the extent of its knowledge is limited by a fixed set of axioms, whereas people can discover unexpected truths [Jones and Wilson, 1995]. Nonetheless we strongly believe in the opposite and suggest that self-reference should be implemented by means of building hierarchically structured organic formalized models, where each level has different plasticity, and components of one level are partially duplicated in the levels above. The proposed structure thus is very similar to the hierarchical neural structure of the brain, while ability to discover unexpected truths found in the brain can be achieved through efficient application of Boltzmann's entropy in information processing [Prigogine and Stengers, 1984].

In genuinely organic models under the term "formalization" we firstly understand ability to interpret. Fresh data just received by the system from the outside, should be assigned connections to the exact data already available to the system. Secondly we understand ability to generate. The system itself should be able to produce new values, production of which is dictated by the inability of the system to carry on with interpretational function. Thirdly it is a recursive process of checking all available information for non contradiction of a re-built system of new connections (interpretation) with newly received values (generation). This is particularly important since every time new data is received, the system's architecture inevitably changes.

2 Entropy and Information Processing

Holy Entropy! It's boiling!
[Gamow, 1965]

2.1 Entropy

There is some kind of mystique about entropy. According to [Denbigh, 1990], [Tribus, 1963], von Neumann suggested to Shannon the use of the word "entropy" adding that "it will give you a great edge in debates because nobody really knows what entropy is anyway". The 2nd law of thermodynamics appears both in thermodynamics and in kinetic theory, in thermodynamics it is postulated, but in kinetic theory 2nd law is introduced by Boltzmann. Boltzmann's entropy, defined as a function of the values taken by the macroscopic variables, equals the logarithm of that volume, where microscopic variables are the positions and the momentum of the particles [Boltzmann, 1974]. This way entropy looks quite arbitrary. We may define as many types of entropy as we can find sets of macroscopic variables since kinetic theory doesn't differentiate micro from macro particles. Boltzmann's theory suggests that the act of collision occurs in such a way that the particles after collision have no "memory" of their precondition and their distribution is not related to speeds and other characteristics of particles before the collision. Integrals of movement (summed energy, impulses, etc) are kept. This way the collision itself is located out with kinetic theory. At the moment of collision particles lose their individuality and importantly, correlations of high orders between variables, used to describe particles, disappear too. If one looks at collision as a process of power interaction between particles, then it is possible to apply Liouville's theorem to the movement of the particles [Ma, 1985]. The theorem states that the phase volume of a system of particles, and respectively entropy (according to Boltzmann) stay unchanged. The equation of kinetic theory, suggests a projection of equations in a phase space of high dimension (dimension is directly proportional to the number of particles) on a simpler phase space [Lebowitz and Penrose, 1973]. In thermodynamics, the standard phase space is the one of Gibbs [Jaynes, 1991]. However if another phase space is taken, then different entropy will be derived. It is because of the fact that entropy can be derived from other phase spaces that we conclude that reached entropy is objective only in terms of corresponding phase space. Jaynes, following Wigner, calls entropy "anthropomorphic" [Jaynes, 1983]. A better word might be "contextual", i.e. entropy depends on the physical situation and on its level of description.

2.2 Information Processing

As mentioned in previous paragraphs, the concept of entropy is highly important, as it is responsible for the generation of new abstracts or "unexpected" truths using only the information available to the system. Thus entropy is closely related to the processing of information.

While it is true that "energy" correlates with the velocity of particles ($e = mv^2$) and that particles with higher energy spend more time in more places than slower particles higher energy particles are better able to surmount and escape barriers which stop slower particles [Lestienne, 1990] . Contrary to the common belief human aspire to maximum entropy rather than minimum [Prigogine, 1996]. In the biological system of the brain processing of information in the form of memory imprints and memorized scenarios is clearly useful mechanism for survival and higher forms of self-fulfillment. This also clearly represents process of reducing entropy in the information processing system of the brain. On the other hand, being able to generate creative new scenarios and fresh solutions out of memorized material in unpredictable external conditions conveyed to the brain by sensory inputs is a contrary process where entropy in the brain's information processing system is increased in order to enable new configurations to emerge.

For use in this article, relation between entropy and methods of processing information is defined as follows:

$$I(Y, X) = H(Y) - H(Y \mid X)$$

I – Information, H – Entropy, X – Input, Y – Output

Importance of information processing is hidden in ability to choose adequate model, describing given information. Universal approach to choosing a model is a method of building a description of minimal (lesser) length.

$$t^*=\mathrm{ARGMIN}\{f(x^N|t):t \text{ in } T\}$$

T – Model's space

f(x^N) – Length of description of a sequence of N realizations of random value x, equal on average to N*H(x), where H(x) is entropy (Shannon's Source Coding Theorem).
Conceive
Accordingly entropy can be described as follows. If the random value x has probability density of P(x), then its entropy is equal to

$$H(x) = -\mathrm{INT}\ [p(x)*\log (p(x))\ dx]$$

There is no code, using which it is possible to transmit any information, consisting of N miscorrelated realizations of random value x with less than N*H(x) bit. Meaning entropy is an average number of bits, needed to describe one realization of a random value. However there always exists a code with random length H(x) + 1 (for one realization). Huffman's algorithm describes such a code. By code reflection of sets of possible meanings of the random value in binary sequence is understood.

Latter suggests that if there is a formalized structure, involved in processing of some information flow, its architecture should be developed through the properties of incoming information flow.

3 Inductive Automaton

As mentioned above in our research we propose building an organic formalized system by means of a hierarchical neural network. We refer to this network as 'inductive automaton' and partially base it on some of the concepts proposed by Emelyanov-Yaroslavsky in 1990 [Emelyanov-Yaroslavsky, 1990]. The main idea was not to implement a specific functionality of each class of tasks but to define one basic task – energy consumption minimization and then to obtain all other functionalities as consequences of it, that is, as by-products.

The second criterion of developing that network was to look for a strong analogy between the artificial network and natural neural systems. Our research largely depends on the field of neurodynamics which has advanced steadily in the past quarter century. Based on the studies of the brain, neurodynamics suggest that brains are hierarchically organized, with multiple levels from quanta to neurons, and yet more levels from neurons to societies of brains [Freeman, 1975]. It also assumes that brain functions can be approached by separation into domains with the independent variables of space, time, and amplitude, thereby avoiding, in first approximations, the complexity of nonlinear, time-varying, non-stationary equations. Entropy, the phenomenon of organized disorder constantly changing with fluctuations across the edge of stability, is not to be discarded and it is directly addressed in this paper.

Researches in artificial intelligence can be divided into two groups: informational and biological approaches. In our research we assume that the combination of those two approaches is the key to success and in this paper we will focus on both perspectives. Inductive automaton, like natural neural networks, has emotional centre that controls levels of activity in network and excitatory links formation. Our opinion is that the explanation of emotional centre function and memory function, presented by Emelyanov-Yaroslavsky [Emelyanov-Yaroslavsky, 1990], is not complete, as there is no mention of entropy which we consider to be definitive in information processing both found in nature and directly related to energy consumption minimization. During his investigation the author is trying to understand bio-logical natural neural networks and different mechanisms observed in natural networks but he is not trying to answer a question: why are all these complicated mechanisms needed? The question arises: is it is possible to use more simple and transparent mechanisms that provide the same intelligence with a little loss of functionality? These questions are not answered in his monograph about inductive automaton. Therefore, there is a need for additional investigation.

4 Memory Function

Proceeding similarly to Emelyanov-Yaroslavsky, we suggest considering memory as a general phenomenon. However in our research we define memory as an organizational component of neurocybernetics. Memory can be observed in various biological systems. We assume memorizing to be a mechanism of self-organization of inductive automaton. We will say that external or internal information flow is memorized by inductive automaton if and only if it is able to reproduce this information flow. This understanding makes it possible to separate a number of layers in the system's structure. We can identify a logical layer consisting of sequences of neuron activations in neural network to external stimuli and previous internal reactions, and a physical layer consisting of connection links between groups of neurons that store and link information and neurons themselves that interpret stored information. It is well known, that neural networks can realize an associative memory function [Pchelkin, 2003]. We assume that the network primarily stores memorized sequences and then only static images. That's why we describe our neurons as being Gnostic.

In Neurocognitive Theory, Gnostic cells are neurons that are capable of possessing memory of something complex (such as the image of someone's face). Associative memory function could be understood as follows: if in situation *S* a neural network has generated reaction *R* (sequence of single activation) then next time in the same situation *S* the probability of the same reaction *R* must increase [Pchelkin, 2003]. Thus, inductive automaton records only such sequences as they are able to reproduce. We consider this property as the key mechanism for the formation of more and more abstract models of external information flow. Therefore, we propose to analyze another point of view on memory – memory is something that can restore information. This helps defining abstractions formation process as a consequence of memorizing unlimited information flow by physically limited in capacity memory storage referred to as long-term memory. Consequently, we can define abstraction as something that helps to restore information without recording it (or by minimal recording). Abstractions can be of N-levels, each further away level having lesser connection with original information. Accordingly when abstraction of N-level evolve (at least 3^{rd} level) they don't have any direct interpretation of original information, and are kept only in verbal form and are used only in context. Here it seems to be interesting to observe hypothesis that ability to verbalize allows human to create abstractions of higher levels.

While process of memorization is done by limited long-term memory, traditionally represented by cognitive neuroscientists in terms of the structure of a neural network's connections, short-term memory is represented in terms of the patterns of activation across the network [Hebb, 1949], [Caianiello, 1961]. However, recent neural-network models of short-term verbal working memory (VWM) have used modifiable structural connections to encode item and order information [Hartley and Houghton, 1996], [Burgess and Hitch, 1999]. In these models, words are stored by changing the connection weights between linguistic units, and phenomena related to VWM are thus modeled with long-term memory structures and mechanisms. Accordingly, the formation of a structure must imply memorizing, and growth of logical layer based on fixed physical layer. This process is possible only when an inductive automaton is able to successfully search for new abstract forms of memorized information. Thus, the formation of a long-term memory structure implies formation of the more abstract model of the external world.

5 EC – Emotional Centre

The main output parameter of a neuron is spike frequency, therefore the transfer function of the neuron is gradual, but unlike a popular sigmoid it has not two, but three stable states: inactive, half active and fully active. After full activation of a neuron occurs, self-locking and forced deactivation of the neuron takes place because full activation is only a temporal state. However, a neuron can maintain half active state for a longer period of time. The half active state of a neuron corresponds to a state of "hi-dissipation" in which energy consumption becomes greater, while the fully active state corresponds to a state of "lo-dissipation" as it allows the economy of energy over the longer period of time. The idea of self-organization is based on these two qualitative states of "hi-dissipation" and "lo-dissipation": the inductive automaton builds new links between neurons in order to make the state of the network better. The state of "hi-dissipation" for the network is defined by the number of half active neurons.

Emelyanov-Yaroslavsky introduces an Emotional Centre (referred to as EC) to make such self-organization more efficient [Emelyanov-Yaroslavsky, 1990]. EC is a mechanism of neural network activity modulation. It is a subset of special neurons. These special neurons control other neurons (Gnostic memory neurons) by two parameters: (1) the shared threshold D (in the original concept [Emelyanov-Yaroslavsky, 1990] this threshold was not distinctly referred – it is only a coefficient of the dynamic threshold of a neuron, this coefficient depends on the state of an EC shared between all neurons) and (2) the reinforcement coefficient that determines proportion of temporal links that transform to persistent links. The first parameter controls the activity of neural network while the second one is self-organizing. EC needs a regular influence from memory neurons. EC and memory neurons are interacting as an oscillating system. In the state of "lo-dissipation" of EC the value of the shared threshold D must increase. It makes more difficult for memory neurons to reach a fully active state. After a very short period of time the state of EC must become worse. But in the state of "hi-dissipation" of EC the value of the shared threshold D must decrease making it easier for memory neurons to reach a fully active state, and resulting in EC switching to the state of "lo-dissipation". The value of the reinforcement coefficient increases only during improvement of the state of EC. Such interrelations between EC and memory neurons promote the setting of the oscillatory process: the state of "lo-dissipation" in EC creates the state of "hi-dissipation" in memory neurons. The state of "hi-dissipation" in memory neurons creates the state of "hi-dissipation" in EC, and the state of "hi-dissipation" in EC activates the network that consecutively reaches the state of "lo-dissipation". During these oscillations the EC controls count of half active neurons.

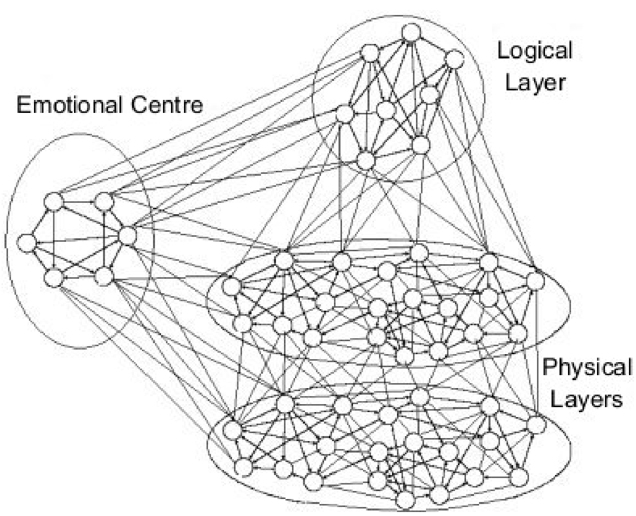
Since an inductive automaton is a self-organizing (with formalization) emulation of a closed physical system, as a consequence it obeys physical laws. Through modeling of a closed physical system, interesting behavior was observed: Boltzmann's entropy of the system was at first increasing but later, with appearance of fluctuations, it decreased to its minimum. This leads to a conclusion that when EC is in the state of "lo-dissipation" the entropy of the system in given phase space is at its minimum. With oscillations continuing, it starts growing till the network neurons reach full activation. At this moment entropy is at its maximum and new combinations of information are produced until physical fluctuations appear and result in a lock-out and respectively change to the state of "lo-dissipation" of the network as energy consumption returns to minimum. Before each activity reaches its maximum half activation occurs (the state of consistent recovery), but after each maximum the deactivation of active neurons occurs, and the activity is transferred to other groups of neurons. Thus network activity maximums correspond to the minimal value of shared threshold D and maximum value if entropy.

6 Hierarchical Neural Network

As stated earlier we propose building an inductive automaton as a hierarchical neural network. Levels of hierarchy are unimodal sensory Gnostic neurons (referred to as memory neurons capable of 'knowing' about something or possessing memory of something), multimodal sensory Gnostic neurons, and abstractions of the first level to the abstractions of n-level. Thus in practical terms for every knowledge unit there exists a network-(1) of neurons. Later neurons of network-(1) come together and form abstract neural network-(2). Further all neurons of abstract network-(2) unite with other neural networks having logical relations with network-(1) and form network-(3) which contains a full description of knowledge unit. The final network-(3) will have a number of inputs corresponding to the number of modalities involved in creation of the full description. Later the network-(3) connects with phase space map and creates elementary neural connection links. This way with evolution of the system we observe an empirical model of the environment, which develops not only by means of connection links of phase space relations but also by the connection links of knowledge unit properties.

Further based upon abstractions of the first level, abstractions of the second level are formed which still preserve connectives with the sensory knowledge units. Abstractions of the third level evolve in a similar way. However they don't have any direct sensory interpretation, and are kept only in verbal form and are used only in context. Knowledge units represented in verbal form are stored in VWM which as mentioned above modeled with long-term memory structures and mechanisms Here it seems to be interesting to observe hypothesis that ability to verbalize allows human to create abstractions of higher levels. Neurons grouped together by connection links make up the full description of a single knowledge unit. Naturally there exist a number of neurons that belong to a number of different groups describing different knowledge units, those neurons can be described as being on the crossing between knowledge units. While sensory connections lead external stimuli to the network-(1) – identifier of knowledge unit, further neurons are responsible only for the process of unification of information creating higher levels of abstractions. Interactions on the level of abstractions lead to procession of high volumes of information, as even a simple abstraction of low level includes a high volume of information.

Here is an illustration of the proposed architecture including physical and logical network layers and Emotional Centre.



As mentioned earlier in the paragraph describing EC, neurons describing knowledge units are normally in the state of inactivity. When knowledge is being retrieved activation (impulse generation) of neurons describing knowledge units involved occurs. Various neuron groups can switch into half active state which triggers connection links between related groups to switch into a working state (not impulse generation). Neurons located on crossings between various neuron groups receive inputs from a number of other groups describing knowledge units (minimum two) and accordingly switch to half active state (impulse generations), this process continues till required knowledge is retrieved and EC sends a signal to put neurons back into the state of inactivity.

7 Conclusions

Biologically inspired approaches and models have already proven useful in various machine intelligence tasks. Neural networks imitate only loosely the interconnectivity and dynamics of biological neurons, but provide nevertheless a powerful new formalism and information processing platform for challenging tasks like pattern recognition and associative memory.

The biological idea of autonomous, self-building and self-sustaining entities is a powerful paradigm that can be utilized in many ways in future AI systems. Biological properties like autonomy and the ability to develop new solutions on the fly become desirable if not obligatory in situations where we send AI systems into distant or hard-to-reach places where instant communication and remote control are not possible. Apart from cognitive and deductive capabilities such systems also need to have a motivational subsystem that keeps them acting in a goal-oriented fashion even in totally new and unexpected situations.

While individual neurons in an artificial neural network may be involved in arbitrarily complex and detailed information processing tasks, they are simultaneously participating in system-wide signaling states characterized by macroscopic variables like total signal count and total system entropy. We propose the modulation of these macroscopic variables as a novel way to implement a motivational feedback loop that would support not only learning and abstraction capabilities but also creative generation of fresh ideas and solutions.

Making the individual neurons motivated to seek given activity levels we generate a system-wide pull towards a steady dynamic state characterized by total signal count and average signaling activity being in a given range. The role of Emotional Centre, the motivational control subsystem in our model, is to push the system further from this steady state in situations where the normal rate of information processing and the existing information-carrying macrostructures in the form of stored patterns and generated abstractions are not enough to solve a given challenge. Assuming a system of limited memory capacity and connectivity, the temporarily applied higher entropy levels translate into intensified breaking down of existing patterns and abstractions and consequent recombination and regeneration of novel patterns and abstractions to be tested on the new situation.

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